CHAPTER 6

MECHANICAL PROPERTIES OF METALS

PROBLEM SOLUTIONS

Concepts of Stress and Strain

6.1 Using mechanics of materials principles (i.e., equations of mechanical equilibrium applied to a free-body diagram), derive Equations 6.4a and 6.4b.

Solution

This problem asks that we derive Equations 6.4a and 6.4b, using mechanics of materials principles. In Figure (a) below is shown a block element of material of cross-sectional area \( A \) that is subjected to a tensile force \( P \). Also represented is a plane that is oriented at an angle \( \theta \) referenced to the plane perpendicular to the tensile axis; the area of this plane is \( A' = A/\cos \theta \). In addition, the forces normal and parallel to this plane are labeled as \( P' \) and \( V' \), respectively. Furthermore, on the left-hand side of this block element are shown force components that are tangential and perpendicular to the inclined plane. In Figure (b) are shown the orientations of the applied stress \( \sigma \), the normal stress to this plane \( \sigma' \), as well as the shear stress \( \tau' \) taken parallel to this inclined plane. In addition, two coordinate axis systems in represented in Figure (c): the primed \( x \) and \( y \) axes are referenced to the inclined plane, whereas the unprimed \( x \) axis is taken parallel to the applied stress.

Normal and shear stresses are defined by Equations 6.1 and 6.3, respectively. However, we now chose to express these stresses in terms (i.e., general terms) of normal and shear forces (\( P \) and \( V \)) as

\[
\sigma = \frac{P}{A}
\]

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\[ \tau = \frac{V}{A} \]

For static equilibrium in the \( x' \) direction the following condition must be met:

\[ \sum F_{x'} = 0 \]

which means that

\[ P' - P \cos \theta = 0 \]

Or that

\[ P' = P \cos \theta \]

Now it is possible to write an expression for the stress \( \sigma' \) in terms of \( P' \) and \( A' \) using the above expression and the relationship between \( A \) and \( A' \) [Figure (a)]:

\[ \sigma' = \frac{P'}{A'} = \frac{P \cos \theta}{A \cos \theta} = \frac{P \cos^2 \theta}{A} = \frac{\sigma}{\cos \theta} \]

However, it is the case that \( P/A = \sigma \); and, after making this substitution into the above expression, we have Equation 6.4a–that is

\[ \sigma' = \sigma \cos^2 \theta \]

Now, for static equilibrium in the \( y' \) direction, it is necessary that

\[ \sum F_{y'} = 0 \]

\[ = -V' + P \sin \theta \]

Or

\[ V' = P \sin \theta \]
We now write an expression for $\tau'$ as

$$\tau' = \frac{V'}{A'}$$

And, substitution of the above equation for $V'$ and also the expression for $A'$ gives

$$\tau' = \frac{V'}{A'} = \frac{P \sin \theta}{A} \frac{A}{\cos \theta} = \frac{P}{A} \sin \theta \cos \theta = \sigma \sin \theta \cos \theta$$

which is just Equation 6.4b.
6.2 (a) Equations 6.4a and 6.4b are expressions for normal (σ') and shear (τ') stresses, respectively, as a function of the applied tensile stress (σ) and the inclination angle of the plane on which these stresses are taken (θ of Figure 6.4). Make a plot on which is presented the orientation parameters of these expressions (i.e., \( \cos^2 \theta \) and \( \sin \theta \cos \theta \)) versus \( \theta \).

(b) From this plot, at what angle of inclination is the normal stress a maximum?

(c) Also, at what inclination angle is the shear stress a maximum?

**Solution**

(a) Below are plotted curves of \( \cos^2 \theta \) (for σ') and \( \sin \theta \cos \theta \) (for τ') versus \( \theta \).

(b) The maximum normal stress occurs at an inclination angle of 0°.

(c) The maximum shear stress occurs at an inclination angle of 45°.
6.4 A cylindrical specimen of a titanium alloy having an elastic modulus of 108 GPa and an original diameter of 3.9 mm will experience only elastic deformation when a tensile load of 2000 N is applied. Compute the maximum length of the specimen before deformation if the maximum allowable elongation is 0.42 mm.

Solution

We are asked to compute the maximum length of a cylindrical titanium alloy specimen (before deformation) that is deformed elastically in tension. For a cylindrical specimen

\[ A_0 = \pi \left( \frac{d_0}{2} \right)^2 \]

where \( d_0 \) is the original diameter. Combining Equations 6.1, 6.2, and 6.5 and solving for \( l_0 \) leads to

\[
l_0 = \frac{\Delta l}{\varepsilon} = \frac{\Delta l}{\sigma} \frac{E}{F} = \frac{\Delta l}{F} \frac{E \pi \left( \frac{d_0}{2} \right)^2}{E_A} = \frac{\Delta l E \pi d_0^2}{4F} \]

\[
= \frac{(0.42 \times 10^{-3} \text{ m}) (108 \times 10^9 \text{ N/m}^2) (\pi) (3.9 \times 10^{-3} \text{ m})^2}{(4)(2000 \text{ N})}
\]

\[ = 0.257 \text{ m} = 257 \text{ mm} \]
6.7 For a bronze alloy, the stress at which plastic deformation begins is 280 MPa, and the modulus of elasticity is 115 GPa.

(a) What is the maximum load that may be applied to a specimen with a cross-sectional area of 325 mm\(^2\) without plastic deformation?

(b) If the original specimen length is 120 mm, what is the maximum length to which it may be stretched without causing plastic deformation?

**Solution**

(a) This portion of the problem calls for a determination of the maximum load that can be applied without plastic deformation \(F_y\). Taking the yield strength to be 280 MPa, and employment of Equation 6.1 leads to

\[
F_y = \sigma_y A_0 = (280 \times 10^6 \text{ N/m}^2)(325 \times 10^{-6} \text{ m}^2)
\]

\[= 91,000 \text{ N}\]

(b) The maximum length to which the sample may be deformed without plastic deformation is determined from Equations 6.2 and 6.5 as

\[
l_f = l_0 \left(1 + \frac{\sigma}{E}\right)
\]

\[= (120 \text{ mm}) \left[1 + \frac{280 \text{ MPa}}{115 \times 10^9 \text{ MPa}}\right] = 120.29 \text{ mm}\]
6.10 Consider a cylindrical specimen of a steel alloy (Figure 6.21) 15.0 mm in diameter and 75 mm long that is pulled in tension. Determine its elongation when a load of 20,000 N is applied.

Solution

This problem asks that we calculate the elongation $\Delta l$ of a specimen of steel the stress-strain behavior of which is shown in Figure 6.21. First it becomes necessary to compute the stress when a load of 20,000 N is applied using Equation 6.1 as

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi \left( \frac{d_0}{2} \right)^2} = \frac{20000 \text{ N}}{\pi \left( \frac{15 \times 10^{-3} \text{ m}}{2} \right)^2} = 113 \text{ MPa}$$

Referring to Figure 6.21, at this stress level we are in the elastic region on the stress-strain curve, which corresponds to a strain of 0.0007. Now, utilization of Equation 6.2 to compute the value of $\Delta l$

$$\Delta l = \varepsilon l_0 = (0.0007)(75 \text{ mm}) = 0.0525 \text{ mm}$$
6.11 Figure 6.22 shows, for a gray cast iron, the tensile engineering stress–strain curve in the elastic region. Determine (a) the tangent modulus at 10.3 MPa, and (b) the secant modulus taken to 6.9 MPa.

Solution

(a) This portion of the problem asks that the tangent modulus be determined for the gray cast iron, the stress-strain behavior of which is shown in Figure 6.22. In the figure below is shown a tangent drawn on the curve at a stress of 10.3 MPa.

The slope of this line (i.e., $\frac{\Delta \sigma}{\Delta \varepsilon}$), the tangent modulus, is computed as follows:

$$\frac{\Delta \sigma}{\Delta \varepsilon} = \frac{15 \text{ MPa} - 5 \text{ MPa}}{0.0074 - 0.0003} = 1410 \text{ MPa} = 1.41 \text{ GPa}$$

(b) The secant modulus taken from the origin is calculated by taking the slope of a secant drawn from the origin through the stress-strain curve at 6.9 MPa. This secant is drawn on the curve shown below:
The slope of this line (i.e., $\frac{\Delta \sigma}{\Delta \varepsilon}$), the secant modulus, is computed as follows:

$$\frac{\Delta \sigma}{\Delta \varepsilon} = \frac{15 \text{ MPa} - 0 \text{ MPa}}{0.0047 - 0} = 3190 \text{ MPa} = 3.19 \text{ GPa}$$
6.13 In Section 2.6 it was noted that the net bonding energy $E_N$ between two isolated positive and negative ions is a function of interionic distance $r$ as follows:

$$E_N = -\frac{A}{r} + \frac{B}{r^n} \quad (6.25)$$

where $A$, $B$, and $n$ are constants for the particular ion pair. Equation 6.25 is also valid for the bonding energy between adjacent ions in solid materials. The modulus of elasticity $E$ is proportional to the slope of the interionic force–separation curve at the equilibrium interionic separation; that is,

$$E \propto \left(\frac{dF}{dr}\right)_{r_0}$$

Derive an expression for the dependence of the modulus of elasticity on these $A$, $B$, and $n$ parameters (for the two-ion system) using the following procedure:

1. Establish a relationship for the force $F$ as a function of $r$, realizing that

$$F = \frac{dE_N}{dr}$$

2. Now take the derivative $dF/dr$.

3. Develop an expression for $r_0$, the equilibrium separation. Since $r_0$ corresponds to the value of $r$ at the minimum of the $E_N$-versus-$r$ curve (Figure 2.8b), take the derivative $dE_N/dr$, set it equal to zero, and solve for $r$, which corresponds to $r_0$.

4. Finally, substitute this expression for $r_0$ into the relationship obtained by taking $dF/dr$.

**Solution**

This problem asks that we derive an expression for the dependence of the modulus of elasticity, $E$, on the parameters $A$, $B$, and $n$ in Equation 6.25. It is first necessary to take $dE_N/dr$ in order to obtain an expression for the force $F$; this is accomplished as follows:

$$F = \frac{dE_N}{dr} = \frac{d}{dr} \left(-\frac{A}{r}\right) + \frac{d}{dr} \left(\frac{B}{r^n}\right)$$

$$= \frac{A}{r^2} - \frac{nB}{r^{n+1}}$$

The second step is to set this $dE_N/dr$ expression equal to zero and then solve for $r$ ($= r_0$). The algebra for this procedure is carried out in Problem 2.14, with the result that
\[ r_0 = \left( \frac{A}{nB} \right)^{1/(1-n)} \]

Next it becomes necessary to take the derivative of the force \((dF/dr)\), which is accomplished as follows:

\[
\frac{dF}{dr} = \frac{d}{dr} \left( \frac{A}{r^n} \right) + \frac{d}{dr} \left( -\frac{nB}{r^{(n+1)}} \right)
\]

\[
= -\frac{2A}{r^3} + \frac{(n)(n+1)B}{r^{(n+2)}}
\]

Now, substitution of the above expression for \(r_0\) into this equation yields

\[
\left( \frac{dF}{dr} \right)_{r_0} = -\frac{2A}{\left( \frac{A}{nB} \right)^{3/(1-n)}} + \frac{(n)(n+1)B}{\left( \frac{A}{nB} \right)^{(n+2)/(1-n)}}
\]

which is the expression to which the modulus of elasticity is proportional.
6.18 A cylindrical specimen of a hypothetical metal alloy is stressed in compression. If its original and final diameters are 20.000 and 20.025 mm, respectively, and its final length is 74.96 mm, compute its original length if the deformation is totally elastic. The elastic and shear moduli for this alloy are 105 GPa and 39.7 GPa, respectively.

Solution

This problem asks that we compute the original length of a cylindrical specimen that is stressed in compression. It is first convenient to compute the lateral strain \( \varepsilon_x \) as

\[
\varepsilon_x = \frac{\Delta d}{d_0} = \frac{20.025 \text{ mm} - 20.000 \text{ mm}}{20.000 \text{ mm}} = 1.25 \times 10^{-3}
\]

In order to determine the longitudinal strain \( \varepsilon_z \) we need Poisson's ratio, which may be computed using Equation 6.9; solving for \( \nu \) yields

\[
\nu = \frac{E}{2G} - 1 = \frac{105 \times 10^3 \text{ MPa}}{(2)(39.7 \times 10^3 \text{ MPa})} - 1 = 0.322
\]

Now \( \varepsilon_z \) may be computed from Equation 6.8 as

\[
\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{1.25 \times 10^{-3}}{0.322} = -3.88 \times 10^{-3}
\]

Now solving for \( l_0 \) using Equation 6.2

\[
l_0 = \frac{l_z}{1 + \varepsilon_x} = \frac{74.96 \text{ mm}}{1 - 3.88 \times 10^{-3}} = 75.25 \text{ mm}
\]
6.23 A cylindrical rod 100 mm long and having a diameter of 10.0 mm is to be deformed using a tensile load of 27,500 N. It must not experience either plastic deformation or a diameter reduction of more than $7.5 \times 10^{-3}$ mm. Of the materials listed as follows, which are possible candidates? Justify your choice(s).

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Yield Strength (MPa)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum alloy</td>
<td>70</td>
<td>200</td>
<td>0.33</td>
</tr>
<tr>
<td>Brass alloy</td>
<td>101</td>
<td>300</td>
<td>0.34</td>
</tr>
<tr>
<td>Steel alloy</td>
<td>207</td>
<td>400</td>
<td>0.30</td>
</tr>
<tr>
<td>Titanium alloy</td>
<td>107</td>
<td>650</td>
<td>0.34</td>
</tr>
</tbody>
</table>

**Solution**

This problem asks that we assess the four alloys relative to the two criteria presented. The first criterion is that the material not experience plastic deformation when the tensile load of 27,500 N is applied; this means that the stress corresponding to this load not exceed the yield strength of the material. Upon computing the stress

\[
\sigma = \frac{F}{A_0} = \frac{F}{\pi \left(\frac{d_0}{2}\right)^2} = \frac{27,500 \text{ N}}{\pi \left(\frac{10 \times 10^{-3} \text{ m}}{2}\right)^2} = 350 \times 10^6 \text{ N/m}^2 = 350 \text{ MPa}
\]

Of the alloys listed, the Ti and steel alloys have yield strengths greater than 350 MPa.

Relative to the second criterion (i.e., that $\Delta d$ be less than $7.5 \times 10^{-3}$ mm), it is necessary to calculate the change in diameter $\Delta d$ for these three alloys. From Equation 6.8

\[
\nu = -\frac{\epsilon}{\epsilon} = -\frac{\Delta d}{\frac{d_0}{\sigma}} = -\frac{E \Delta d}{\sigma d_0}
\]

Now, solving for $\Delta d$ from this expression,

\[
\Delta d = -\frac{\nu \sigma d_0}{E}
\]
For the steel alloy

$$\Delta d = - \frac{(0.30)(350 \text{ MPa})(10 \text{ mm})}{207 \times 10^3 \text{ MPa}} = -5.1 \times 10^{-3} \text{ mm}$$

Therefore, the steel is a candidate.

For the Ti alloy

$$\Delta d = - \frac{(0.34)(350 \text{ MPa})(10 \text{ mm})}{107 \times 10^3 \text{ MPa}} = -11.1 \times 10^{-3} \text{ mm}$$

Hence, the titanium alloy is not a candidate.
Tensile Properties

6.25 Figure 6.21 shows the tensile engineering stress–strain behavior for a steel alloy.

(a) What is the modulus of elasticity?
(b) What is the proportional limit?
(c) What is the yield strength at a strain offset of 0.002?
(d) What is the tensile strength?

Solution

Using the stress–strain plot for a steel alloy (Figure 6.21), we are asked to determine several of its mechanical characteristics.

(a) The elastic modulus is just the slope of the initial linear portion of the curve; or, from the inset and using Equation 6.10

\[ E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{(200 - 0) \text{ MPa}}{(0.0010 - 0)} = 200 \times 10^3 \text{ MPa} = 200 \text{ GPa} \]

The value given in Table 6.1 is 207 GPa.

(b) The proportional limit is the stress level at which linearity of the stress–strain curve ends, which is approximately 300 MPa.

(c) The 0.002 strain offset line intersects the stress–strain curve at approximately 400 MPa.

(d) The tensile strength (the maximum on the curve) is approximately 515 MPa.
6.29 A cylindrical specimen of aluminum having a diameter of 12.8 mm and a gauge length of 50.800 mm is pulled in tension. Use the load–elongation characteristics shown in the following table to complete parts (a) through (f).

<table>
<thead>
<tr>
<th>Load N</th>
<th>Length mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50.800</td>
</tr>
<tr>
<td>7,330</td>
<td>50.851</td>
</tr>
<tr>
<td>15,100</td>
<td>50.902</td>
</tr>
<tr>
<td>23,100</td>
<td>50.952</td>
</tr>
<tr>
<td>30,400</td>
<td>51.003</td>
</tr>
<tr>
<td>34,400</td>
<td>51.054</td>
</tr>
<tr>
<td>38,400</td>
<td>51.308</td>
</tr>
<tr>
<td>41,300</td>
<td>51.816</td>
</tr>
<tr>
<td>44,800</td>
<td>52.832</td>
</tr>
<tr>
<td>46,200</td>
<td>53.848</td>
</tr>
<tr>
<td>47,300</td>
<td>54.864</td>
</tr>
<tr>
<td>47,500</td>
<td>55.880</td>
</tr>
<tr>
<td>46,100</td>
<td>56.896</td>
</tr>
<tr>
<td>44,800</td>
<td>57.658</td>
</tr>
<tr>
<td>42,600</td>
<td>58.420</td>
</tr>
<tr>
<td>36,400</td>
<td>59.182</td>
</tr>
</tbody>
</table>

Fracture

(a) Plot the data as engineering stress versus engineering strain.
(b) Compute the modulus of elasticity.
(c) Determine the yield strength at a strain offset of 0.002.
(d) Determine the tensile strength of this alloy.
(e) What is the approximate ductility, in percent elongation?
(f) Compute the modulus of resilience.

Solution

This problem calls for us to make a stress–strain plot for aluminum, given its tensile load–length data, and then to determine some of its mechanical characteristics.
(a) The data are plotted below on two plots: the first corresponds to the entire stress–strain curve, while for the second, the curve extends to just beyond the elastic region of deformation.
(b) The elastic modulus is the slope in the linear elastic region (Equation 6.10) as

\[ E = \frac{\Delta \sigma}{\Delta \varepsilon} = \frac{200 \text{ MPa} - 0 \text{ MPa}}{0.0032 - 0} = 62.5 \times 10^3 \text{ MPa} = 62.5 \text{ GPa} \]

(c) For the yield strength, the 0.002 strain offset line is drawn dashed. It intersects the stress–strain curve at approximately 285 MPa.

(d) The tensile strength is approximately 370 MPa, corresponding to the maximum stress on the complete stress-strain plot.
(e) The ductility, in percent elongation, is just the plastic strain at fracture, multiplied by one-hundred. The total fracture strain at fracture is 0.165; subtracting out the elastic strain (which is about 0.005) leaves a plastic strain of 0.160. Thus, the ductility is about 16% EL.

(f) From Equation 6.14, the modulus of resilience is just

\[ U_r = \frac{\sigma^2}{2E} \]

which, using data computed above gives a value of

\[ U_r = \frac{(285 \text{ MPa})^2}{2 (62.5 \times 10^3 \text{ MPa})} = 0.65 \text{ MN/m}^2 = 0.65 \times 10^6 \text{ N/m}^2 = 6.5 \times 10^5 \text{ J/m}^3 \]
True Stress and Strain

6.39 Show that Equations 6.18a and 6.18b are valid when there is no volume change during deformation.

Solution

To show that Equation 6.18a is valid, we must first rearrange Equation 6.17 as

\[ A_i = \frac{A_0}{l_0} \]

Substituting this expression into Equation 6.15 yields

\[ \sigma_T = \frac{F}{A_i} = \frac{F}{\frac{A_0}{l_0}} = \sigma \left( \frac{l}{l_0} \right) \]

But, from Equation 6.2

\[ \epsilon = \frac{l}{l_0} - 1 \]

Or

\[ \frac{l}{l_0} = \epsilon + 1 \]

Thus,

\[ \sigma_T = \sigma \left( \frac{l}{l_0} \right) = \sigma ( \epsilon + 1 ) \]

For Equation 6.18b

\[ \epsilon_T = \ln (1 + \epsilon) \]

is valid since, from Equation 6.16

\[ \epsilon_T = \ln \left( \frac{l}{l_0} \right) \]

and

\[ \frac{l}{l_0} = \epsilon + 1 \]

from above.
6.40 Demonstrate that Equation 6.16, the expression defining true strain, may also be represented by

\[ \varepsilon_t = \ln \left( \frac{A_0}{A} \right) \]

when specimen volume remains constant during deformation. Which of these two expressions is more valid during necking? Why?

**Solution**

This problem asks us to demonstrate that true strain may also be represented by

\[ \varepsilon_t = \ln \left( \frac{A_0}{A} \right) \]

Rearrangement of Equation 6.17 leads to

\[ \frac{l}{l_0} = \frac{A_0}{A_i} \]

Thus, Equation 6.16 takes the form

\[ \varepsilon_t = \ln \left( \frac{l}{l_0} \right) = \ln \left( \frac{A_0}{A_i} \right) \]

The expression \( \varepsilon_t = \ln \left( \frac{A_0}{A} \right) \) is more valid during necking because \( A_i \) is taken as the area of the neck.
The following true stresses produce the corresponding true plastic strains for a brass alloy:

<table>
<thead>
<tr>
<th>True Stress MPa</th>
<th>True Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>345</td>
<td>0.10</td>
</tr>
<tr>
<td>415</td>
<td>0.20</td>
</tr>
</tbody>
</table>

What true stress is necessary to produce a true plastic strain of 0.25?

Solution

For this problem, we are given two values of $\varepsilon_T$ and $\sigma_T$, from which we are asked to calculate the true stress which produces a true plastic strain of 0.25. Employing Equation 6.19, we may set up two simultaneous equations with two unknowns (the unknowns being $K$ and $n$), as

$$\log (345 \text{ MPa}) = \log K + n \log (0.10)$$

$$\log (415 \text{ MPa}) = \log K + n \log (0.20)$$

Solving for $n$ from these two expressions yields

$$n = \frac{\log (345) - \log (415)}{\log (0.10) - \log (0.20)} = 0.266$$

and for $K$

$$\log K = 4.96 \text{ or } K = 10^{4.96} = 630 \text{ MPa}$$

Thus, for $\varepsilon_T = 0.25$

$$\sigma_T = K (\varepsilon_T)^n = (630 \text{ MPa})(0.25)^{0.26} = 440 \text{ MPa}$$
6.46 Find the toughness (or energy to cause fracture) for a metal that experiences both elastic and plastic deformation. Assume Equation 6.5 for elastic deformation, that the modulus of elasticity is 172 GPa, and that elastic deformation terminates at a strain of 0.01. For plastic deformation, assume that the relationship between stress and strain is described by Equation 6.19, in which the values for \( K \) and \( n \) are 6900 MPa and 0.30, respectively. Furthermore, plastic deformation occurs between strain values of 0.01 and 0.75, at which point fracture occurs.

**Solution**

This problem calls for us to compute the toughness (or energy to cause fracture). The easiest way to do this is to integrate both elastic and plastic regions, and then add them together.

\[
\text{Toughness} = \int \sigma \, d \varepsilon
\]

\[
= \int_0^{0.01} E \varepsilon \, d \varepsilon + \int_0^{0.75} K \varepsilon^p \, d \varepsilon
\]

\[
= \frac{E}{2} \varepsilon^2 \bigg|_0^{0.01} + \frac{K}{(n+1)} \varepsilon^{(n+1)} \bigg|_0^{0.75}
\]

\[
= \frac{172 \times 10^9 \text{ N/m}^2}{2} (0.01)^2 + \frac{6900 \times 10^6 \text{ N/m}^2}{(1.0 + 0.3)} \left[ (0.75)^{1.3} - (0.01)^{1.3} \right]
\]

\[
= 3.65 \times 10^9 \text{ J/m}^3
\]
6.47 For a tensile test, it can be demonstrated that necking begins when

\[
\frac{d\sigma_T}{d\varepsilon_T} = \sigma_T
\]  \hspace{1cm} (6.26)

Using Equation 6.19, determine the value of the true strain at this onset of necking.

Solution

Let us take the derivative of Equation 6.19, set it equal to \(\sigma_T\), and then solve for \(\varepsilon_T\) from the resulting expression. Thus

\[
\frac{d\left[K(\varepsilon_T)^n\right]}{d\varepsilon_T} = Kn(\varepsilon_T)^{(n-1)} = \sigma_T
\]

However, from Equation 6.19, \(\sigma_T = K(\varepsilon_T)^n\), which, when substituted into the above expression, yields

\[
Kn(\varepsilon_T)^{(n-1)} = K(\varepsilon_T)^n
\]

Now solving for \(\varepsilon_T\) from this equation leads to

\[
\varepsilon_T = n
\]

as the value of the true strain at the onset of necking.
6.51 (a) A 10-mm-diameter Brinell hardness indenter produced an indentation 1.62 mm in diameter in a steel alloy when a load of 500 kg was used. Compute the HB of this material.

(b) What will be the diameter of an indentation to yield a hardness of 450 HB when a 500 kg load is used?

**Solution**

(a) We are asked to compute the Brinell hardness for the given indentation. It is necessary to use the equation in Table 6.5 for HB, where \( P = 500 \) kg, \( d = 1.62 \) mm, and \( D = 10 \) mm. Thus, the Brinell hardness is computed as

$$
HB = \frac{2P}{\pi D \left[ D - \sqrt{D^2 - d^2} \right]}
$$

$$
= \frac{(2)(500 \text{ kg})}{(\pi)(10 \text{ mm}) \left[ 10 \text{ mm} - \sqrt{(10 \text{ mm})^2 - (1.62 \text{ mm})^2} \right]} = 241
$$

(b) This part of the problem calls for us to determine the indentation diameter \( d \) which will yield a 450 HB when \( P = 500 \) kg. Solving for \( d \) from the equation in Table 6.5 gives

$$
d = \sqrt{D^2 - \left[ D - \frac{2P}{(HB)\pi D} \right]^2}
$$

$$
= \sqrt{(10 \text{ mm})^2 - \left[ 10 \text{ mm} - \frac{(2)(500 \text{ kg})}{(450)(\pi)(10 \text{ mm})} \right]^2} = 1.19 \text{ mm}
$$
6.52 Estimate the Brinell and Rockwell hardnesses for the following:

(a) The naval brass for which the stress–strain behavior is shown in Figure 6.12.

(b) The steel alloy for which the stress–strain behavior is shown in Figure 6.21.

**Solution**

This problem calls for estimations of Brinell and Rockwell hardnesses.

(a) For the brass specimen, the stress–strain behavior for which is shown in Figure 6.12, the tensile strength is 450 MPa. From Figure 6.19, the hardness for brass corresponding to this tensile strength is about 125 HB or 70 HRB.

(b) The steel alloy (Figure 6.21) has a tensile strength of about 515 MPa [Problem 6.25(d)]. This corresponds to a hardness of about 160 HB or ~90 HRB from the line for steels in Figure 6.19. Alternately, using Equation 6.20a

\[
\text{HB} = \frac{7\times(\text{MPa})}{3.45} = \frac{515 \text{ MPa}}{3.45} = 149
\]