and hence show that $|\sum_{x} x^2| \leq |x - \frac{1}{x}|$.

Now reason that

$\sum_{x} x^2 = |x - \frac{1}{x}|$

that value, pick one, $x^2$ the real that $\mathbf{V} = x^2$ to show

absolute value is $x$. If two of more equations have this
cleared with $x$. Suppose the element of $x$, having largest

value of $x$ and let $E$, be an eigenvector. Then let $x$ be an eigenv.

Prove Corollary's theorem. Hence let $x$ be an eigenv.

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7. Let $S$ consist of all vectors in $R^n$ of partial to the

real equal to the sixth component.

16. $S$ consists of all vectors in $R^n$ of whose first component

is the sum of the first six components.

15. $S$ consists of all vectors in $R^n$ whose second component

is equal to $2x$.

14. $S$ consists of all vectors in $R^n$ having third component

consistently are equal.

13. $S$ consists of all vectors in $R^n$ having zero third and

second

12. $S$ consists of all vectors in $R^n$. Having zero third and

second

11. $S$ consists of all vectors in $R^n$.

10. $S$ consists of all vectors in $R^n$.

9. $S$ consists of all vectors in $R^n$.

8. $S$ consists of all vectors in $R^n$.

7. $S$ consists of all vectors in $R^n$.

In each of Problems 1 through 16 determine whether the

set of vectors is a subspace of $R^n$. For the applicable n.
Let $A$ be an $n \times n$ matrix. Then $\text{rank}(A) = n$ if and only if $A_R = I_n$.

**Proof** If $A_R = I_n$, then the number of nonzero rows in $A_R$ is $n$, since $I_n$ has no zero rows. Hence in this case $\text{rank}(A) = n$.

Conversely, suppose that $\text{rank}(A) = n$. Then $A_R$ has $n$ nonzero rows, hence no zero rows. By definition of a reduced matrix, each row of $A_R$ has leading entry 1. Since each row, being a nonzero row, has a leading entry, then the $i, i$ elements of $A_R$ are all equal to 1. But it is also required that if column $j$ contains a leading entry, then all other elements of that column are zero. Thus $A_R$ must have each $i$, $j$ element equal to zero if $i \neq j$, so $A_R = I_n$. ■

**Example 6.24**

Let

$$A = \begin{pmatrix} 1 & -1 & 4 & 2 \\ 0 & 1 & 3 & 2 \\ 3 & -2 & 15 & 8 \end{pmatrix}.$$  

We find that

$$A_R = \begin{pmatrix} 1 & 0 & 7 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$  

Therefore, $\text{rank}(A) = 2$. This is also the dimension of the row space of $A$ and of the column space of $A$. ■

In the next section we will use the reduced form of a matrix to solve homogeneous systems of linear algebraic equations.

**Section 6.4 Problems**

In each of Problems 1 through 20, (a) find the reduced form of the matrix, and from this the rank, (b) find a basis for the row space of the matrix, and the dimension of this space, and (c) find a basis for the column space, and the dimension of this space.

1. $\begin{pmatrix} -4 & 1 & 3 \\ 2 & 2 & 0 \end{pmatrix}$

2. $\begin{pmatrix} 1 & -1 & 4 \\ 0 & 1 & 3 \\ 2 & -1 & 11 \end{pmatrix}$

3. $\begin{pmatrix} -3 & 1 \\ 2 & 2 \\ 4 & -3 \end{pmatrix}$

4. $\begin{pmatrix} 6 & 0 & 0 & 1 & 1 \\ 12 & 0 & 0 & 2 & 2 \\ 1 & -1 & 0 & 0 & 0 \end{pmatrix}$

5. $\begin{pmatrix} 8 & -4 & 3 & 2 \\ 1 & -1 & 1 & 0 \end{pmatrix}$

6. $\begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Gram-Schmidt orthonormalization process: Use this process to orthonormalize the group of vectors:

a) \( \mathbf{v}^1 = (1, 1) \), \( \mathbf{v}^2 = (2, 1) \)

b) \( \mathbf{v}^1 = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \), \( \mathbf{v}^2 = \left( \begin{array}{c} 1 \\ 1 \end{array} \right) \), \( \mathbf{v}^3 = \left( \begin{array}{c} 0 \\ 1/2 \end{array} \right) \)

Matrix Norm: Compute the matrix norm: \( \mathbf{L}_1(A), \mathbf{L}_2(A), \mathbf{L}_\infty(A) \):

da) \( A = \left( \begin{array}{cc} 1 & 2 \\ 2 & 8 \end{array} \right) \), b) \( \left( \begin{array}{ccc} -2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & -1 \end{array} \right) \)

c) \( \left( \begin{array}{ccc} 2 & 1 & 0 \\ 1 & -2 & 4 \\ 0 & 4 & 2 \end{array} \right) \)