Boundary element method for vibration analysis of two-dimensional anisotropic elastic solids containing holes, cracks or interfaces

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\begin{abstract}
By using the anisotropic elastostatic fundamental solutions and employing the dual reciprocity method, a special boundary element method (BEM) was developed in this paper to perform elastodynamic analysis of anisotropic elastic plates containing holes, cracks or interfaces. The system of ordinary differential equations obtained for the vibration transient analysis was solved using the Houlbolt’s algorithm and modal superposition method. These equations were reduced to the standard eigenproblem for free vibration, and a purely algebraic system of equations for steady-state forced vibration. Since the fundamental solutions used in the present BEM satisfy the boundary conditions set on the holes, cracks, or interfaces, no meshes are needed along these boundaries. With this special feature, the numerical examples presented in this paper show that to get an accurate result much fewer elements were used in the present BEM comparing with those in the traditional BEM or finite element method.
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1. Introduction

Stress concentration induced by holes, cracks, or interfaces is usually the main cause for the local failure of a structure. Although analytical closed form solutions have been found long time ago for an anisotropic plate containing an elliptical hole, crack or interface [1–6], they are applicable only for an infinite domain with static loads. In the design of engineering structures, numerical simulations such as the finite element method and boundary element method, play an important role to deal with the problems of complicated structural geometry and complex static or dynamic loading conditions. To increase the accuracy and efficiency of the numerical simulation, suitable utilization of the existing analytical solutions sometimes will make a great improvement, which motivates the present study.

For the traditional boundary element method (BEM) of dynamic analysis, the fundamental solutions used in boundary integral equation (BIE) are based on the solutions of the dynamic body force, i.e., a unit impulse force, which is in terms of time variable. However, practically it is difficult to find the solution analytically for particular problems, such as anisotropic plates with cracks, holes, or interfaces subjected to an impulse force. Even though such fundamental solutions could be obtained, the domain integral would have been present in the BIE if the body forces or non-zero initial conditions do exist [7]. In view of this, Nardini and Brebbia [8] developed the dual reciprocity boundary element method (DRBEM) to deal with the problems which involve the domain integral. By using a series of distributed shape functions to approximate the source term, the domain integral can be transformed into the equivalent boundary integral numerically. The applications of this method have been successful in a wide range of fields [9]. Thus, the elastostatics fundamental solutions can be used for dynamic analysis if the inertia term of dynamic problems is treated as a general body force, which is the source term remained in the domain integral.

A lot of approximate functions, such as general radial basis function (RBF), spline, multiquadric and Gaussian types RBFs, have been discussed to increase the accuracy of DRBEM [10]. For the isotropic elastic problems the most commonly used function is the first order conical radial basis function. However, in the case of anisotropic materials the particular solutions derived by the functions described above are not easy to be coped with in closed form. Kögl and Gaul [11,12] suggested an alternative approach to choose a particular solution, which is employed in this study and will be discussed in detail in the related section. Recently, the dual reciprocity method and the radial integration method have also been successfully used to treat the domain integrals of shear deformable orthotropic cracked plates under transverse distributed loads [13].

By the benefit of dual reciprocity method, the boundary element method for vibration analysis of anisotropic plates...