In-plane/out-of-plane concentrated forces and moments on composite laminates with elliptical elastic inclusions

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Abstract

The problems of composite laminates containing elliptical elastic inclusions subjected to concentrated forces and moments are considered in this paper. By employing Stroh-like formalism for the coupled stretching–bending analysis, analytical closed form solutions are obtained explicitly. The generality of the solutions provided in this paper can be shown as follows: (1) The laminates include any kinds of laminate lay-ups, symmetric or unsymmetric, which allow the stretching and bending deformations couple each other. (2) The concentrated forces and moments can be applied in in-plane and/or out-of-plane directions, located inside and/or outside the inclusions. (3) The elliptical elastic inclusions can be any kinds of elastic materials including the limiting cases such as holes, rigid inclusions, cracks, line inclusions, etc. Since no such general solution has been found in the literature, the solutions are checked and verified by the special cases that no inclusions are embedded in the laminates, and that the inclusions are replaced by holes. Moreover, with various hardness ratios of inclusion and matrix some numerical examples showing the stress resultants along the interface are presented. Like the Green’s functions for the infinite laminates and those containing holes/cracks, the present solutions associated with the in-plane concentrated forces and out-of-plane concentrated moments have exactly the same mathematical form as those of the corresponding two-dimensional problems, in which the only difference is the contents of the symbols. While for the other loading cases, new types of solutions are obtained explicitly.

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1. Introduction

On account of the linear character of the related equations, the principle of superposition is applicable to most of the fundamental problems of elasticity. Thus, the solutions associated with the concentrated forces and moments, generally called Green’s functions, become important in constructing general solutions through superposition. Because of its importance, many analytical solutions of Green’s functions have been published