Explicit solutions for the coupled stretching–bending problems of holes in composite laminates

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Abstract

Although the classical lamination theory was developed long time ago, it is still not easy to apply this theory to find the analytical solutions for the curvilinear boundary value problems especially when the stretching and bending are coupled each other. To overcome the difficulties, recently we developed a Stroh-like formalism for the general composite laminates. By using this formalism, most of the relations for the coupled stretching–bending problems can be organized into the forms of Stroh formalism for two-dimensional anisotropic elasticity problems. With this newly developed Stroh-like formalism, it becomes easier to obtain an analytical solution for the coupled stretching–bending problems of holes in composite laminates. Because the Stroh-like formalism is a complex variable formalism, the analytical solutions for the whole field are expressed in complex form. Through the use of some identities derived in this paper, the resultant forces and moments around the hole boundary are obtained explicitly in real form. Due to the lack of analytical solutions for the general cases, the comparison is made with the existing analytical solutions for some special cases. In addition, to show the generality of our analytical solutions, several numerical examples are presented to discuss the coupling effect of the laminates and the shape effect of the holes.

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1. Introduction

The problems of composite laminates containing holes have been studied extensively for two-dimensional problems. Although the classical lamination theory was developed long time ago (Jones, 1974), it is not easy to apply this theory to find the analytical solutions for the problems with curvilinear boundaries, like the hole problems. Even the problems of composite laminates with holes are very important in engineering applications and have been solved vastly in two-dimensional problems, it is rarely solved when the laminates make the in-plane and plate bending problems couple each other subjected to in-plane forces and/or out-of-plane bending moments. Searching for the literature, the only