CONTROL OF GLOBAL INSTABILITY IN A NON-PARALLEL NEAR WAKE

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ABSTRACT

The effect of base suction on a plane wake was found to produce significant changes on wake dynamics. The wake is produced by merging two boundary layers from the trailing edge of a splitter plate in a two stream water tunnel. A threshold suction speed exists which is equal to half of the free stream velocity. If the suction speed is below the threshold, the wake flow is unstable. If the suction speed is above the threshold, the wake becomes stable and no vortex shedding is observed. In the present experiment, the suction technique can stabilize a wake at a Reynolds number of 2,000.

The suction significantly reduces the length of the absolutely unstable region in the immediate vicinity of the trailing edge of the splitter plate and produces a converging flow pattern, resulting in the breakdown of global instability. The global growth rate changes from positive (unstable) to negative (stable) at the suction speed equaling 0.46 of the free stream velocity. The threshold suction speed can be accurately predicted by the global linear theory of Monkewitz et al. with non-parallel flow correction.
I. INTRODUCTION

Vortex shedding which appears downstream from bluff bodies above a critical Reynolds number has been studied since the turn of this century. Vortex shedding induces drag, generates noise and causes structure vibration. During the past few decades, several techniques have been developed to control vortex shedding. Roshko (1955) showed that a thin partition placed along the centerline of the wake could prevent the shedding of Karman vortices and reduce the drag. By using blowing at the trailing edge, Wood (1964, 1967) reduced the strength of vortices.

Recently, the concept of the absolute and convective instabilities (Huerre & Monkewitz 1985, 1990) has provided new physical insights to wake control. The difference between absolute instability and convective instability is that the impulse response of a fluid system can propagate in both upstream and downstream directions in an absolutely unstable flow, whereas it can only propagate in the downstream direction in a convectively unstable flow region. Therefore, a flow system that contains a sufficiently large region of absolute instability will respond to external forcing by developing time-amplifying global oscillations, a response that is fundamentally different from that of a system that is convectively unstable everywhere. Both types of instabilities exist in the wake flow. The near-wake is governed by absolute instability. The wake then changes to convective instability at a short distance from the solid boundary due to the rapid filling of the velocity deficit. In the absolutely unstable region, wake flow acts as a resonator where all unstable disturbances are self-excited or time-amplified. In the convectively unstable region, wake flow is similar to a spatial amplifier in that all unstable frequencies will grow exponentially along the downstream direction (for small excitation...
amplitudes). Therefore, the disturbances will first resonate in the absolutely unstable region and then serve as an initial perturbation in the convectively unstable region.

Numerical and experimental evidence indicates that the wake instabilities are strongly influenced by an absolutely unstable region in the near-wake. This fact suggests the possibility of controlling global flow characteristics through modification of the streamwise velocity profiles at the origin of wake flow. Some methods for controlling Karman vortex shedding in the near wake have been proposed. In the following, we briefly review methods for controlling Karman vortex shedding. Wood (1964, 1967) showed the effects of reducing the strength of individual vortices by blowing at the trailing edge. Bearman (1967) also demonstrated that sufficient base bleed leads to a reduction of base drag. Hannemmann & Oertel (1989) successfully suppressed vortex shedding from bluff bodies by bleeding fluid from the blunt base in their numerical simulation. Strykowski & Sreenivasan (1990) placed a small control cylinder with a diameter of typically 1/8 to 1/20 of the primary cylinder diameter in the near wake of the primary cylinder and suppressed vortex shedding at a Reynolds number of about 80. The most effective location for the suppression of vortex shedding is generally in the shear layers around the mean recirculation region. The effect of the control cylinder is speculated to be the breaking of the mean flow symmetry and cancellation of vorticity which are responsible for the reduction of absolute instability. Another well-known method for suppression of vortex shedding from bluff bodies at a low Reynolds number is through heat addition to the near wake. The connection of this effect to local stability properties was investigated by Yu & Monkewitz (1990). The primary purpose of heating was to decrease local absolute growth rates by reducing the fluid density of the near wake. The above mentioned studies have shown that Karman vortex shedding can be greatly manipulated by a small modification in the near-wake. However, most of these control techniques are effective only at a low Reynolds number. In the present study, the
wake flow with base suction control provides an approach to wake flow control at a Reynolds number of 2,000. The wake becomes globally stable by the suction control (Leu and Ho 1992). This experimental finding is consistent with the numerical simulation works of Hammond and Redekopp (1995) where a nondimensional threshold suction speed of 0.4 is reported at Re=160.

The paper is based on Leu's Ph.D. thesis (1994). The flow visualization experiments are first used to present the remarkable change in a wake flow with suction control. Afterwards, the threshold suction speed is determined and the stability characteristics of a wake flow are identified by experimental methods. The changes of the near-wake flow field from subcritical to supercritical suction are examined. Finally, the prediction of the threshold suction speed by global stability analysis and the description of a wake flow dynamics by the Stuart-Landau equation are presented.
II. EXPERIMENTAL FACILITIES AND INSTRUMENTATION

The experiments were performed in an open surface water channel (Figure 1). The channel was designed for studies of wake/mixing layer. The stagnation chamber is separated into two compartments by a vertical splitter plate. Flow in each compartment is supplied by a pump. Each pump is controlled by a valve. The velocity ratio between the two free streams can be easily adjusted by these two valves. The water channel includes three parts: a stagnation chamber, a splitter plate and a test section. Water is pumped from the reservoir into the stagnation chambers. A perforated plate, honeycomb and six layers of screens are used to improve the mean flow uniformity and to reduce the turbulence level. The splitter plate is placed vertically to avoid the troublesome problem of removing air bubbles from the screens. The test section is equipped with a glass window on top to eliminate the free surface wave. The width and height of the test section of the water channel is 40.0 and 20.0 cm respectively. The maximum freestream velocity $U_\infty$ is 20 cm/sec. The typical turbulence level in the test section is about 0.6% of the time averaged velocity. At the end of the water channel, two 4 inch diameter PVC pipes serve as return passages to the upstream reservoir. The trailing edge of the splitter plate is a suction slot with the width $D = 1.0$ cm. Throughout the paper the Reynolds number is based on the freestream velocity $U_\infty$ and the width of the suction slot $D$. The spanwise dimension of the slot is 20.0 cm. Inside the hollow splitter plate region, a perforated tube is connected to a third pump. The third pump provides suction at the trailing edge of the splitter plate. Two fine mesh screens and a plastic honeycomb section made of stacks of straws are placed between the perforated tube and suction slot to reduce the turbulence level and the non-uniformity of suction in the spanwise direction. The suction speed $U_s$ is defined as the maximum absolute value of the negative suction velocity at $X=0, Y=0$ location. The turbulence level is about 1.0% of the time averaged suction speed at the suction slot. The non-uniformity of suction in the
spanwise direction is less than 2% of the time averaged suction speed. The configuration of the trailing edge and coordinate system are shown in Figure 1.

The flow is visualized by using hydrogen bubbles. The velocity field is obtained by a one-component DANTEC 55X Laser Doppler Anemometer (LDA) system. The system is set up in forward scatter mode and equipped with a Bragg cell frequency shifter for reversing flow detection. The water channel is seeded with fluorescent coating plastic particles with sizes ranging from 20 to 50 µm. The scattering light from the particles passing through the measuring volume is received by a photomultiplier tube (PMT). The PMT signals are analyzed by TSI IFA550 frequency analyzer.

The entire LDA optical system is mounted on a three-dimensional traversing mechanism which is controlled by an 80386 personal computer. The analog output of TSI IFA550 frequency analyzer is digitized by Data Translation DT2801A analog-to-digital (A/D) converter. The fastest digitizing rate is 27.5 KHz. In all cases, the digitizing frequency is about two orders of magnitude higher than the vortex shedding frequency. The U velocity in the X-direction and the V velocity in the Y-direction (Figure 1) are measured by rotating the LDA system by 90 degrees. By measuring the U and V velocities of the cross section in the X-Y plane with very fine grids, the spanwise vorticity, $\omega_z$, is obtained by the difference of $dV/dX$ and $dU/dY$ whereby the partial derivatives are calculated from the central difference of the U and V time-average velocity fields.

A forcing device consisting of a rod and a vibrator is used to investigate the stability of the wake flow. The forcing device vibrates the rod periodically with a sinusoidal motion. The control rod is placed at the neutral position location of $X/D=0.5$, $Y/D=0$. The phase of the control rod is determined by the output of a phase indicator. The phase indicator consists of a magnetic micro switch (Honeywell 3AV2C) and a ferrous vane blade. The
vane blade rotates with the stepping motor. The stepping motor is controlled by a 80286 personal computer. The computer sends the desired number of pulses in an exact time period. Therefore, the frequency of the oscillating rod and the number of oscillating cycles can be controlled.
III: GENERAL FEATURES OF A WAKE WITH SUCTION

3.1 Flow Visualization

The wake flow controlled by suction is first examined by using hydrogen bubbles in the water channel. The bubbles serve as flow tracers convected in the streamwise direction which is from top to bottom in Figures 2 and 3. When suction is not applied, the flow from both streams merge at the trailing edge of the splitter plate. The flow is unstable due to the velocity shear. Karman vortex street can be observed in Figure 2. The Reynolds number is about 700, which is based on the free stream velocity, $U_\infty$, and the width of the suction slot, D. After the suction is applied, the flow is still unstable at low suction speeds. The suction speed $U_s$ defined as the maximum absolute value of the negative velocity is measured by LDA at $X/D=0$, $Y/D=0$ location. While the suction speed, $U_s$, increases and reaches a threshold level, a different type of wake flow occurs; the periodic vortices are not observed (Fig. 3). The stable wake flow seems to persist in the streamwise direction. The suction speed at which the wake flow can be stabilized is defined as the threshold suction speed, $U_{TS}$. The threshold suction speed equals about one half of the free stream speed of a wake flow (Leu & Ho 1993). The accurate way of defining $U_{TS}$ will be discussed in Chapter V.

3.2 Velocity Signals

Figure 4 displays the time traces of the streamwise velocity inside the shear layer at the location $X/D=5.0$, $Y/D=1.0$ for four different suction speeds. In the wake flow without suction, the velocity time trace (Fig. 4a) shows an almost periodic sinusoidal wave pattern which corresponds to the passage of vortices. As suction speed $U_s/U_\infty=0.24$ is applied at the trailing edge of a splitter plate, the velocity time trace (Fig. 4b) still shows
similar periodic fluctuations but not so regular. As long as the suction speed $U_s$ is less than the threshold, $U_{Ts}$, the flow is periodic in time. However, once the suction speed $U_s/U_\infty$ is greater than the threshold, the time trace at suction speed $U_s/U_\infty=0.68$ (Fig. 4c) shows an almost constant velocity trace. The signal indicates that the flow becomes globally stable which confirms the flow visualization results.

From the spectrum in an unstable wake flow, the dominant frequency, i.e., the frequency with the highest amplitude in the energy spectra represents the vortex shedding frequency in a wake flow (Fig. 5). When the suction speed is higher than the threshold velocity, $U_s > U_{Ts}$, the flow is stabilized. However, the background noise in the water channel still produces disturbances to drive the flow. This is clear from the velocity trace shown in Fig. 4c which displays the intermittent disturbances. The spectra (Fig. 5) are broadband due to the intermittent pulse-type fluctuations. This problem prevented us from performing a clean stability study.

### 3.3 The Role of a Stabilizing Rod

We found that a thin rod with a dimension of 1 mm in diameter placed downstream of the suction slot ($X/D=0.5$, $Y/D=0$) is able to further stabilize the wake flow. The effects of the thin rod are demonstrated by the velocity trace (Fig. 4d) and the energy spectra (Fig. 5). The flow becomes very stable and the velocity remains constant for all time. The spectrum is at least two orders in magnitude lower than the case without a stabilizing rod at the same suction speed. The almost flat energy spectrum indicates that the disturbances have been reduced to the electronic noise level associated with the instrument.
The stabilizing rod is very helpful since most of the experiments in the absolute/convective instability studies suffered from the signal-to-noise ratio problem. Continuous background noise creates difficulty in identifying a fluid system as either an absolute or convective instability. The control rod enables us to identify a flow system in a very "lean" environment.

**IV: VELOCITY FIELD OF THE NEAR WAKE**

**4-1 The Flow Reversal Region**

In free shear flows, the streamwise development is very sensitive to the initial conditions (Ho and Huerre 1984). In the present case, the suction certainly affects the flow near the origin and therefore changes the global feature. In this section, the time-averaged streamwise and transverse velocity components, $U$ and $V$, in the near field of the suction slot are reported. Velocity components at 900 survey stations in the center plane of the water channel ($Z/D=10$) were obtained by Laser Doppler Anemometer (LDA). The velocity vectors in the case of the wake flow of free stream velocity $U_\infty=10.0$ cm/s with threshold suction, $U_s/U_\infty=0.5$, are shown in Figure 6. The Reynolds number is 1050. Two sharp edge marks at -5.0 mm and 5.0 mm of the Y-coordinate indicate the suction slot location. A region of reversed flow extending approximately 1 cm from the trailing edge is observed. Inside the flow reversal region, there exists two standing eddies. By using $U$ and $V$ velocity in Figure 6, one can calculate the streamlines by a mass balance. The corresponding streamline pattern is plotted in Figure 7. A saddle point was found in the streamline pattern. The dividing streamline which separates the reversed flow region from the outer flow region shows that the fluid being sucked into the slot is approximately 3.0 mm in thickness from each side of the slot. A strong two-dimensional
flow around the saddle point is noticed. This feature raises a concern about the validity of the parallel flow assumption used in the stability analyses. Hence, non-parallel flow effects have to be considered in the present studies.

4-2 The Location of Saddle Point

A reversed flow region suggests the existence of an absolute instability in the wake flow (Huerre and Monkewitz 1985,1990). The extent of the absolutely unstable region can be indicated by the distance from the suction slot to the saddle point. The position of the saddle point is obviously a function of the suction speed. However, the method used in Figures 6 and 7 for determining the saddle point position is cumbersome. Since the wake flow velocity profile is symmetric, an alternative method is used. The streamwise velocity distributions along the centerline (Y/D=0) of the wake with free stream velocity $U_\infty=10.0 \text{ cm/s}$ are shown in Figure 8. The typical centerline velocity distribution is a negative velocity region followed by a positive velocity region. The zero crossing point is the saddle point location. At zero suction, the negative velocity is very small and the saddle point is located about 30 mm downstream from the suction slot. When the suction speed is increased, the centerline velocity changes from a negative value to a positive value within a short distance. The zero crossing point, i.e. the saddle point, moves closer to the trailing edge. The locations of the saddle points, $X_s$, at different suction velocities, are shown in Figure 9. As the suction is applied, the change in the saddle point location is very large. When the suction is close to the threshold speed, the position of the saddle point is about one slot width downstream from the suction slot. For further increases in the suction speed, the position of the saddle position moves upstream but at a much slower rate.
Since the position of the saddle point can approximately represent the length of the absolutely unstable region, the absolutely unstable region will reduce its size with the increase of suction. Chomaz, Huerre and Redekopp (1990) suggested that the absolute instability region must have a sufficient size in order to sustain a global mode. Therefore, it is plausible that the absolutely unstable region may be reduced in such a way that the global mode may no longer exist. On the other hand, the temporal growth rates of streamwise velocity profiles will increase with increasing suction. Whether global instability will exist or not depends on the relative effects of these two factors.

4-3 The Initial Boundary Layer Velocity Profiles

The streamline pattern in Figure 7 shows that most of the fluid sucked into the slot comes from a thin layer near the solid boundary. It suggests that the boundary layer must be significantly modified by the suction process. Furthermore, the vorticity which drives the instability almost resides entirely inside the layer. Therefore, it should be interesting to investigate the effects on the inflow boundary layers caused by the suction.

Time-averaged boundary layer velocity profiles at the trailing edge are measured at four different suction speeds and are shown in Figure 10. The boundary layer velocity profile without suction is first compared with Blasius boundary layer velocity profile. The comparison confirms that no streamwise gradients exist in the free stream when the suction is turned off. The velocity inside the boundary layer is increased by the suction as compensation for the mass depletion through the slot. The distribution of the additional velocity caused by suction can be obtained by subtracting the boundary layer velocity profile without suction from the boundary layer velocity profiles with suction. Wall-jet type velocity profiles are observed (Fig. 11a). The transverse region affected by
the suction extends to about twice as thick as that of the natural boundary layer thickness. These wall-jet type profiles (Schlichting 1979) can be collapsed into a non-dimensional curve (Fig. 11b). The velocity scale used in the non-dimensional plot is the maximum velocity, \( U_{\text{max}} \), in the wall-jet type profile. The transverse length scale, \( Y_{\text{max}}/2 \), is the distance between the wall and the position where the velocity equals one half of the \( U_{\text{max}} \). Therefore, the boundary layer velocity profile with suction can be decomposed into the zero pressure gradient Blasius velocity profile and the wall jet profile under the favorable pressure gradient caused by the suction.

### 4-4 The Vorticity Field

The instability in a velocity shear region is mainly driven by vorticity. It is then interesting to know the vorticity distribution affected by the suction. Based upon the time-averaged velocities, \( U \) and \( V \), at the threshold suction speed in Figure 6, it is easy to obtain the time-averaged vorticity field. The vorticity distribution and the streamline pattern are plotted together in Figure 12. The magnitude of the vorticity represented by the colored bar is normalized by the free stream velocity, \( U_{\infty} \), and the width of slot, \( D \). It is evident that the region with the highest vorticity is located inside the dividing streamlines (Fig. 12a). Since all of the vorticities which drive the instability are produced from the solid walls if there exists a pressure gradient, it is easy to calculate the vorticity production from the boundary layer velocity profiles in Figure 10. In Figure 7, it is also noticed that the vorticity contaminated fluid in the boundary layer will be removed by the suction. Following this idea, one may postulate that the stable wake with threshold suction can be described as the balance of vorticity production on the solid walls and its removal by the suction. For illustrative purposes of this postulation, residual vorticity ratio is defined as
Residual Vorticity Ratio = \frac{\text{vorticity outside the divided streamline with suction } U_s}{\text{vorticity production when } U_s = 0}

The residual vorticity outside the dividing streamline starts to decrease from 100% to about 20% of the vorticity contained in the boundary layers of the wake flow without suction, when the suction speed reaches the threshold level (Fig. 12b).

V: THE THRESHOLD SUCTION SPEED AND GLOBAL STABILITY

5-1 Quantitative Determination of the Threshold Suction Speed

Based upon flow visualization and the velocity traces (Fig. 2 - 4), the wake becomes stable when the suction velocity is higher than about half of the free stream velocity. Here, we will establish a quantitative definition for the threshold suction speed.

Figure 4(a)-(d) display the time traces of streamwise velocity at the location X/D=5.0, Y/D=1.0 with freestream velocity U_∞ = 14.0 cm/sec. The velocity traces change from a continuous periodic signal to an intermittent spiky fluctuation and finally to an almost constant value as the suction increases. A method to determine the threshold speed from the intermittent nature of the signal was developed. A short time-averaged fluctuation level (SFL) of the velocity signal similar to the VITA signal processing scheme (Blackwelder and Kaplan 1976) is defined as:

$$SFL(t) = \left\{ \frac{1}{T} \int_{t-T/2}^{t+T/2} [U(t) - \overline{U}(t)]^2 \, dt \right\}^{1/2}$$

where T is the period of the vortex passage without suction and \( \overline{U} \) is a short time-averaged velocity, defined as:
The profile of SFL(t) clearly indicates the intermittency of the signal. In Figs. 4a and 4d, the signals have almost constant amplitudes. The values of SFL(t) are fairly constant. The SFL(t) for Figs. 4b and 4c does show the intermittent feature.

The value of SFL indicates the stability of the flow. In the tested range, SFL(t) varies from about 0.1 cm/sec (Fig. 4d) to 4 cm/sec (Fig. 4a). In the Fig. 4d case, the value of SFL(t) is much lower than that of Fig. 4a-c. Note that the scale is changed and plotted on the right hand side of Fig. 4d. The probability of SFL(t) being less than a reference value is defined as:

\[
P(U_r) = \text{Prob}(SFL \leq U_r) = \lim_{T \to \infty} \frac{\sum_{i=1}^{k} \Delta t_i}{T}
\]

where \( \Delta t_i \) is the time interval in which SFL \( \leq U_r \) over a long time history record (\( T = 10240T \) in this experiment). The probability distributions at various suction speeds are shown in Fig. 13. When the value of \( U_r \) changes from zero cm/sec to 4 cm/sec, \( P(U_r) \) increases from 0 and approaches 1. \( U_{rc} \) is defined as the value of \( U_r \) at which \( P(U_r) \) reaches the asymptotic value. In other words, the value of \( U_{rc} \) indicates the maximum value of SFL. When the stabilizing rod is not placed (the "without rod" cases in Fig. 13), the value of \( U_{rc} \) initially increases from 2.5 cm/sec to 3.5 cm/sec with increasing suction speed from \( U_s/U_\infty = 0 \) to 0.24. This indicates that in this Reynolds number small suction favors instability. The velocity time trace in Figure 4b and the linear global growth rates in Figure 19 all confirm this feature. As soon as \( U_s/U_\infty \geq 0.49 \), the probability of low velocity fluctuation signals starts to build up. For example, \( P(U_r=1.0 \text{ cm/sec}) \) increases to 40% at \( U_s/U_\infty = 0.49 \) and 70% at \( U_s/U_\infty = 0.69 \) in Fig. 13.
When the stabilizing rod is used (the "ith rod" cases in Fig. 13), \( U_{rc} \) is found to be only about 0.3 cm/sec in the all stable wake flows with a suction speed higher than 50% of the freestream velocity. Hence, \( U_{rc} = 0.3 \text{ cm/sec} \) is taken as a stringent criterion for determining whether the wake is stable or not. We define a stable intermittency function \( I(U_s/U_\infty) \) based on \( U_{rc} \) which measures the percentage of time in which the wake is stable:

\[
I \left( \frac{U_s}{U_\infty} \right) = \text{Prob} \left[ SFL(t) \leq U_{rc} \right] = \lim_{\tau \to \infty} \frac{\sum_{i=1}^{k} \Delta t_i}{\tau}
\]

where \( \Delta t_i \) is the time interval in which \( SFL(t) \leq U_{rc} \). If the value of \( I(U_s/U_\infty) \) is high, it means that the value of \( SFL \) is low and the wake is stable. A sharp rise of \( I(U_s/U_\infty) \) is observed as the suction speed reaches 50% of the freestream velocity in Fig. 14 which confirms the flow visualization results (Fig. 3). This suction speed is defined as the threshold suction speed, \( U_{Ts} \).

5-2 Local Stability Analysis

To gain a further in-depth understanding of the wake stabilized by suction, it should be helpful to resort to the hydrodynamic stability analysis. In principle, the linear hydrodynamic stability analysis in a shear flow starts with a solution of the steady flow equations, which is the time-average flow field in the wake flow. One then considers this solution with small perturbation superimposed, and enquires whether this perturbation grows or decays as time passes. Monkewitz (1988) investigated the viscous effect on the hydrodynamic instabilities in the low Reynolds number two dimensional wake flow. Monkewitz found that the flow is practically inviscid when the Reynolds number is higher than 100. The definition of Reynolds number in the works by Monkewitz is based on the average mean velocity \( U_{1/2} = (U_{Y=0} + U_\infty) / 2 \) and local half width \( b_{1/2} \) of the wake.
which is defined by $U(b^{1/2}) = U_{b^{1/2}}$. In the present experiments, the range of Reynolds number (based on the same velocity scales $U^{1/2}$ and length scale $b^{1/2}$) is from 150 to 650. Therefore, the inviscid Rayleigh equation is adequate.

$$
(U - \frac{\omega}{k})\left(\frac{d^2\Phi}{dy^2} - k^2\Phi\right) - \frac{d^2U}{dy^2}\Phi = 0 \tag{3}
$$

where $U(y)$ is the steady mean velocity profile, $\omega$ is frequency, $k$ is wave number, and $\Phi$ is defined in the form of normal modes of perturbation stream function

$$
\Psi(x, y, t) = \Phi(y) e^{i(kx-\omega t)}.
$$

The Rayleigh equation involves the second derivative of the mean velocity profile. Thus, an accurate fit of the velocity profile is critical. Here, the experimentally measured velocity profiles are curve-fitted with a two-parameter ($R, N$) equation (Monkewitz & Nguyen 1987).

$$
U(y) = 1-R+2RF(y) \tag{4a}
$$

where

$$
R = \frac{U_{Y=0} - U_x}{U_{Y=0} + U_x} \tag{4b}
$$

$$
F(y) = [1 + \sinh^2N(ysinh^{-1}1)]^{-1} \tag{4c}.
$$

The terms in Eq(4a) are made nondimensional with the the average mean velocity $U_{1/2} = (U_{Y=0} + U_{\infty})/2$ and local half width $b_{1/2}$ of the wake, which is defined by $U(b_{1/2}) = U_{1/2}$.

The mean velocity profiles for the wake flow at $Re=1584$ with non-dimensional suction speeds, $U_s/U_x = 0, 0.19, 0.4, 0.5$ and 0.62, at several downstream locations and their corresponding best fitting parameters ($R, N$) are shown in Figure 15. The velocity profiles are first

Different values of $R$ and $N$ are chosen to fit the wake velocity profiles at each streamwise location.
In the wake flow without suction (Fig. 15a), the velocity profile starts at N=2.0 and R=-1.004 right behind the body and approaches N=1.4 and R=-0.492 at X/D=4.4. The slightly reverse flow region (i.e., R < -1 region) was noticed in the region between X/D=0.0 and 2.4. In the wake with suction (Figs 15b,c,d,e), the profiles are all bell-shaped and the shape parameter N does not change very much. The typical values are between N=2.0 just behind the trailing edge and N=1.0 further downstream. The large decrease in the value of R at the initial velocity profiles from R = -1.435 (Fig. 15b) to R=-4.061 (Fig. 15e) is noticed and indicates the increasingly reversed flow due to the increasing suction.

The maximum deviation between the two-parameter curve fitting and the experimental velocity profile is less than 1% in all measurements. The slight overshoot on the shoulder region of the velocity profile is the main cause of these deviations. Monkewitz (1988) studied the overshoot effects on the stability analysis. The variation of eigenvalues |ω₀| and |k₀| is less than 3% for the overshoot, $U_\infty/U_{max}$, between 1.0 and 0.85. In the present case, the maximum overshoot $U_\infty/U_{max}$ is about 0.9. Therefore, the effect on eigenvalues should be less than 3%.

The absolute frequency, $\omega_0$, was calculated by solving the Rayleigh equation with boundary conditions

$$\begin{bmatrix} \phi \\ \phi' \end{bmatrix} (y \to \infty) = \begin{bmatrix} 1 \\ -k \end{bmatrix} e^{ky} \tag{5a}$$

and

$$\begin{bmatrix} \phi \\ \phi' \end{bmatrix} (y \to 0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{continuous mode} \tag{5b}.$$
We have calculated the complex wave number, $k_0$, which has a zero group velocity $\frac{d\omega}{dk}(k_0)=0$ and its corresponding absolute frequency $\omega_0=\omega(k_0)$. The dispersion relation was found by a shooting method. For a given real $k$ and a proper initial guess $\omega$, the equation (3) with boundary conditions of equations (5a) and (5b) is integrated from $y = +\infty$ to $y = y_c$, and from $y = 0$ to $y = y_c$ by using a standard 4th/5th order Runge-Kutta scheme. If the initial guess is an eigenvalue, the solutions from both sides will match at $y=y_c$. Otherwise, the frequency $\omega$ is updated by Newton method until the matching criterion is satisfied. Then, the saddle point $\frac{d\omega}{dk}(k_0)=0$ in the complex $k$-plane can be found by the Newton method. The maximum principle in complex analysis, which states that the maximum and minimum must appear on the boundary, automatically excludes the possibility of finding a maximum or minimum.

Fig. 16a shows the temporal growth rate $\omega^{(D)}_{oi}$ as a function of the streamwise coordinate, $X/D$, at different suction speeds. $\omega^{(D)}_{oi}$ is non-dimensionalized through the use of the freestream velocity $U_\infty$ and the width of the suction slot $D$. The location where $\omega^{(D)}_{oi}$ varies from positive to negative values indicate a change from absolute instability to convective instability.

The maximum temporal growth rate $\omega^{(D)}_{oi}$ becomes larger when the suction speed increases. This is expected due to the increased back flow induced by the suction. The real part of $\omega_0$, $\omega^{(D)}_{or}$, at each downstream location is shown in Figure 16b. The maximum $\omega^{(D)}_{oi}$ as a function of the suction speed is shown in Fig. 17. No clear change of the trend is observed around $U_{Ts}$.
It is known that local absolute instability is a necessary but not a sufficient condition for global instability (Chomaz et al 1988). Chomaz, Huerre and Redekopp (1990) proposed a global instability criterion for slowly varying base flow:

\[
\text{GIC} = \frac{x_a}{D} \int_{0}^{x_a/D} \sqrt{\omega_{oi}^{(D)}} \, dx \geq 0 \quad (1)
\]

\( x_a \) is the position where \( \omega_{oi} \) changes from positive to negative values. \( x_a \) decreases with increasing suction speed (Fig. 16a). Based upon the data in Figs. 16 and 17, GIC is calculated (Fig. 18). GIC smoothly decreases with increasing suction speeds. Again, there is no clear indication of a globally stable flow near \( U_{Ts} \).

### 5.3 The Global Instability Analysis with Non-Parallel Flow Correction

In the case of a parallel flow, the temporal frequency \( \omega_o \) should remain the same along the streamwise location. Hence, the variations of the absolute frequency \( \omega_o \) along the streamwise direction indicate the non-parallel characteristics especially in a wake with suction. The wake flow without suction can be approximated as being parallel, two parameters \( R \) and \( N \) of the mean velocity profiles at different streamwise locations in Figure 15(a) change in such a way as to keep the absolute growth rate, \( \omega_{oi} \), constant with increasing \( X/D \). Figure 16 shows the fairly flat distribution along the streamwise locations from \( X/D = 0 \) to \( X/D = 3 \) at \( U_s = 0 \) case. When the suction increases, the maximum absolute growth rate increases and the zone with constant \( \omega_{oi} \) shortens. At the same time, the slopes of absolute growth rate along the streamwise direction become steeper with the increasing suction speeds, i.e., the flow becomes more non-parallel with the increasing suction speed.

By applying a global linear stability analysis derived for weakly non-parallel shear flow, Monkewitz, Huerre & Chomaz (1993) showed that the non-parallel correction term is
determined by the slope of the absolute frequency $\omega_0(X^t)$ at the dominant turning point, $X^t$, of the WKBJ approximation. The region around the turning point, $X^t$, can be viewed as a wave-maker for the entire flow field. The global frequency of the entire flow field is not locally determined but is instead determined by a region near the turning point. The global frequency, $\omega_G$, can be split into the dominant absolute frequency $\omega_0$ at the turning point, $X^t$, i.e., the frequency of the mode with zero group velocity at turning point, $X^t$, and a small correction term, $\omega_\varepsilon$, corresponding to the non-parallel effect. The global mode frequency for the non-parallel flow is given by:

$$\omega_G \sim \omega_0 + \omega_\varepsilon = \omega_0 + \varepsilon^{2/3} \left\{ \omega_0 (2\omega^t_{X}/\omega^t_{kk})^{-1/3} a_0 \right\}$$

where $a_0 (=2.338)$ is a zero of the Airy function and $X = \varepsilon x$ is the ‘flow’ coordinate in the terminology of the method of multiple scales (Bender & Orszag 1978). The parameter

$$\varepsilon \equiv \lambda_{typ} \left\{ \delta^{-1}(x) [d\delta/dx] \right\}_{typ} << 1$$

characterizes the degree of the spatial inhomogeneity of the basic flow by providing a measure of the change of the typical cross-stream length scale $\delta(x)$ over one typical instability wavelength $\lambda_{typ}$. Monkewitz et al. also found that the turning point in the wake flow is at the origin of the wake, $X^t=0$. Hence, the non-parallel correction term strongly depends on the initial slope of the wake flow at $X/D=0$ in Figures 16a & b. The zero initial slope of the natural wake without suction indicates that the parallel flow assumption can be applied. As the suction speed, $U_s/U_\infty$, increases, the negative slope of the absolute growth rate $\omega_0^{(D)}$ in Figure 16a and the positive slope of $\omega_{or}^{(D)}$ in Figure 16b no longer remains negligible. The non-parallel correction term, $\omega_\varepsilon$, increases significantly.
In deciding the non-parallel correction term, $\omega_{G_2}$ of equation (6a), Equation (6a) is first re-nondimensionalized by using the width of the suction slot D and the freestream velocity $U_\infty$ and becomes

$$\omega_G^{(D)} \sim \omega_o^{1(D)} + a_0 \left\{ \omega_x^{(D)} \left( 2 \frac{\omega_x^{(D)}}{\omega_{kk}^{(D)}} \right)^{-1/3} \right\}$$

(6c)

where the superscript (D) is non-dimensionalized by using the freestream velocity $U_\infty$ and the width of the suction slot D. In equation (6c), the parameter $\varepsilon$ vanishes. The $\omega_{kk}^{(D)}$ term is second derivative of the absolute frequency $\omega_o^{(D)}$ in the complex k-plane which has been computed to find the saddle point $\frac{d\omega}{dk}(k_o) = 0$ in the complex k-plane by using the Newton iteration method. The $\omega_x^{(D)}$ term is the first derivative of the absolute frequency $\omega_o^{(D)}$ at the turing point $X_t=0$ with respect to streamwise coordinate X/D which can be obtained by 2nd-order accurate finite difference approximation at the location X/D=0 in Figure 16. For a typical mean velocity profile of the wake flow with suction in Figure 15, the velocity ratio parameter R varies from -1 ($U_s/U_\infty=0$) to -4 ($U_s/U_\infty=0.62$); the shape parameter changes from a top hat shape (N=2) to a far-wake profile (N=1). In comparison with the absolute frequency $\omega_o^{(D)}$ in Figure 16 and the R and N parameters of the local profiles in Figure 15, the absolute frequency $\omega_o^{(D)}$, especially the temporal growth rate $\omega_{oi}^{(D)}$, was found to be more sensitive to the velocity ratio parameter R than the shape parameter N. Therefore, the $\omega_x^{(D)}$ in the nonparallel correction term of equation (6c) is very sensitive to the change of the velocity ratio parameter R around the location X/D=0, which is defined from the centerline velocity distribution measurement along the streamwise direction near the suction slot. The accuracy of the parameter R is estimated to be about ±0.0125 for the present experimental setup. Hence, the computed value error from the parameter R is very
insignificant with respect to the change of the parameter R in the wake flow with suction. The error is most from ...

The results of the global growth rate through the use of Equation 6(c) are shown in Figure 19. The ascending curve with open circle symbols which are all positive shows that the flow is locally absolutely unstable based upon the leading order of the global growth rate at the turning point, $\omega_{oi}^{(D)} (X'=0)$. If the non-parallel correction term is considered in equation (6c), the global growth rate changes its sign from positive to negative when the suction speed is equal to about 0.46 of the free stream velocity. In other words, the global instability breaks down and the flow becomes stable at the suction speed equaling 0.46 of the free stream velocity $U_\infty$ which is very close to the threshold suction speed $0.5U_\infty$ determined experimentally (Sections 3-1 and 5-1). This study has clearly illustrated the effect of non-parallel flow on global instability.

5-4 Self-excitation of the Wake

Wake is a flow self-excited by the oscillations produced by absolute instability. This concept can be best illustrated by a transient experiment in the present case where the absolute instability can be turned on or off through the non-parallel mechanism.

In the experiment, suction is not applied to the wake flow during the first 6 seconds (Fig. 20). The non-parallel effect is not strong and the near wake is absolutely unstable so that large amplitude velocity fluctuations exist in the flow. At the instant of $t = 6.0$ sec, a supercritical suction speed (i.e., $U_c > U_{Ts}$) is switched on. In two seconds, the non-parallel
flow region is established. The growth rate of the absolute instability becomes less than zero. The velocity trace changes from a large-amplitude oscillations to a nearly flat signal. This experiment has clearly demonstrated that the wake flow can be globally stable if the self-excitation is "hut off" by the non-parallel flow caused by suction in the near wake region.

5-5 The Impulse Response in a Globally Stable Flow

When the absolute instability region is greatly affected by the non-parallel near wake region produced by suction, it will be interesting to examine the response of the stabilized flow to artificial perturbations. It is known that background noise can obscure the experiment involving global instabilities (Huerre & Monkewitz, 1990). We therefore employed the stabilizing rod which can significantly reduce the background noise (Fig. 5). At the same time, the rod can be vibrated by a driving mechanism at a chosen frequency (Leu 1994) and serve as a forcing device to provide the required perturbations.

The rod is driven in an impulse mode. Each impulse contains a train of five cycle oscillations at a frequency of 1.7 Hz which is close to the most amplified frequency of the wake. The response of the flow system to the pulse train is measured by laser Doppler anemometer at different streamwise locations along Y/D=0.5 line. The signal is phase-averaged. Phase reference is based on the electronic signal which drives the oscillating rod. Slight phase jitters between the phase reference and the velocity fluctuations were observed.

The pulse train response at the location X/D=3.0, Y/D=0.5 is shown in figure 21 when the suction speed U_s/U_∞=0.75 is applied at the trailing edge. More than five cycle oscillations with non-constant amplitude are observed. This results from the combined
response of the solid rod and the flow to the original driving signal. The wake is absolutely stable at this suction speed. Figure 22 shows the spatio-temporal evolution of the pulse train in the connective stability region. The convective nature is obvious. The convection speed is 12.5 cm/sec which is 0.78 of the free stream speed. A slight growth of the signal is shown from the first measuring station to the second station. The amplitude of the phase-averaged signal decays slowly which is believed to be caused by the phase jitters between the driving signal and the velocity pulse train arriving at the measuring station. We therefore introduce a different technique in the following section to clarify this phenomenon.

5-6 The Convective Neutrally Stable Flow

For perturbations applied in a steady manner, the evolution of narrow band energy at a specific frequency \( f \) across the velocity shear region, \( E[u'(f)] \), can be used to indicate the growth or decay of the perturbations along the streamwise direction (Ho and Huang 1982). \( E[u'(f)] \) is based on the spectral analysis which is not sensitive to the phase jitters.

\[
E[u'(f)] = \int_{-\infty}^{\infty} [u'(f)]^2 dY
\]

where \([u'(f)]^2\) is the narrow band energy at the frequency \( f \) and can be evaluated from the energy spectrum \( G[f] \) at the location \((X,Y)\) by the following equation.

\[
[u'(f)]^2 = \int_{f-df}^{f+df} G(f) df
\]

where \(df\) equals to 0.031hz.

When the suction is not applied, the spatial variation of \( E[u'(f)] \) at the unforced fundamental frequency is shown in Fig. 23. Exponential growth of the disturbances is
observed as expected. In the other case, a supercritical suction, $U_s/U_\infty = 0.75$, stabilized
the wake. $E[u'(f)]$ has very small amplification rate initially and then keeps almost at a
constant level. The non-parallel flow caused by suction not only stabilizes the absolute
instability but also significantly affects the downstream wake region. The wake flow
becomes convectively neutrally stable.

VI: THE SELECTION OF INSTABILITY FREQUENCY

6-1 Instability Frequency and the Selection Criteria

From an early work on the examination of a plane wake after a sharp trailing of a flat
plate (Masselin and Ho 1985), it was found that the wake vortex passage frequency is the
same as the one predicted by the linear spatial stability analysis based upon the velocity
profile right at the sharp trailing edge of the splitter plate. The frequencies calculated
from downstream velocity profiles do not fit the measured value. After the absolute-
convective instability concept was introduced (Huerre & Monkewitz 1985), the physical
mechanism governing the above-mentioned phenomenon became clear. The unstable
wake is self-excited by the absolute instability in the near wake region. In the case of a
wake developed from a sharp trailing edge, the absolute instability region is very short.
Hence the frequency is likely to be determined by the velocity profile at the trailing edge.
In the present experiment, a finite absolutely unstable zone exists. When the suction
speed is below the threshold suction, the length of the absolutely unstable region and the
amplification rate of the instability vary with the magnitude of the suction. The
instability frequency must be a function of the suction speed. The question is how the global frequency is selected.

Several researchers have investigated the selection mechanism of the instability frequency in a wake flow (Pierrehumbert 1984, Koch 1985, Monkewitz & Nguyen 1987). They have suggested different criteria for the frequency selection process, based on parallel flow analysis. Pierrehumbert (1984) argued that flow is dominated by the fastest growing resonance between the downstream and the upstream instability wave. The global response is determined by the maximum growth rate location, $X_p$, as shown schematically in Figure 24. Therefore, the measured frequency is predicted to be equal to the real part of the branch point frequency at the location $X_p$, $\omega_{or}(X_p)$.

Koch (1985), on the other hand, suggests that the transition point in the flow where both range point frequency is real acts as an effective “reflector” for the instability waves of that particular frequency. Then, the global response is dominated by a local resonance occurring at the transition from the locally absolute to the locally convective instability. Hence, the global response is determined by the absolute frequency at the transition point $X_k$ where $\omega_{or}(X_k)=0$ which is shown in Figure 24. In the paper by Hannemann & Oertel (1989), the global frequency from their numerical simulations agrees well with the local absolute frequency predicted by Koch criterion. Oertel (1990) in his review paper showed that the von Karman vortex street is still dominated by the locally hydrodynamic resonance in the absolutely unstable region at the supercritical Reynolds number, even though the Karman vortex street is in a nonlinear saturated state at that Reynolds number.

Monkewitz & Nguyen (1987) proposed the “initial resonance criterion”. This criterion suggests that the (non-parallel) wake is dominated by the first local resonance with a
non-negative absolute growth rate encountered by the flow. Monkewitz & Nguyen compared this criterion with many experimental results. The predicted frequency agreed with the measured frequency. Therefore, they concluded that the vortex shedding frequency is determined from the most upstream absolutely unstable location, $X_I$ (Fig. 24).

From our experimental data, the instability frequency selected by the initial resonance criterion (Monkewitz & Nguyen 1987) can be obtained by plotting the frequency $\omega_{or}$ at the location $X/D=0$ for different suction speeds (Fig.16b). The predicted frequencies are shown in Figure 26. According to Figure 16a, the maximum absolute growth rate occurs at $X/D = 0$. Therefore, the global frequency based on Pierrehumbert's suggestion is the same as that predicted from the initial resonance criterion. From the same sets of data (Fig.16), the predicted frequency according to Koch's criterion can be obtained from the real part of the transition point frequency, $\omega_{or}(X_k)$, at $\omega_{oi}(X_k)=0$. The experimentally determined frequencies at various suction speeds are also plotted in Fig. 25. The measured frequencies agree with Pierrehumbert's criterion as well as with the initial resonance criterion (Monkewitz & Nguyen 1987).

6-2 Non-linear Analysis of Frequency Selection

The instability waves initially amplify exponentially at the linear growth rate, $\omega_{Gi}$. At large times, the instability evolves into a nonlinear instability regime (Stuart 1971) and
experiences nonlinear saturation. In a flow with an extended absolute instability region coupled with a convective unstable region, it would be interesting to examine the effect of non-linearity. The dynamics of a weakly non-linear flow is described by the Stuart-Landau equation:

$$\frac{dA}{dt} = \left(\omega_{Gi} - i \omega_{Gr}\right)(U_s)A - [l_r + i l_i] |A|^2 A + O(|A|^5)$$  \hspace{1cm} (7)

where $A$ is the amplitude level, $\omega_{Gi}$ is the linear global growth rate, $\omega_{Gr}$ is the corresponding frequency, and $l$ is the Landau constant. Taylor expansion $\omega_{Gi}, \omega_{Gi}$ and the saturation frequency $\omega_{sat}$ near the threshold suction speed $U_{Ts}$, the following approximations can be applied:

$$\omega_{Gi} = \left[ \frac{d\omega_{Gi}}{dU_s}(U_{Ts}) \right] \left[ U_s - U_{Ts} \right] = \alpha \left[ U_{Ts} - U_s \right]$$ \hspace{1cm} (8a)

$$\omega_{Gr} - \omega_{sat} = \left[ \frac{d\omega_{Gr}}{dU_s}(U_{Ts}) \right] - \left[ \frac{d\omega_{Gr}}{dU_s}(U_{Ts}) \right] \left[ U_s - U_{Ts} \right] = \beta \left[ U_s - U_{Ts} \right]$$ \hspace{1cm} (8b)

$$l_r + i l_i = [l_r + i l_i] (U_s)$$ \hspace{1cm} (8c)

where one can obtain Eq(8b) by subtracting two Taylor expansion equations of $\omega_{Gr}$ and $\omega_{sat}$ and eliminating the high order terms since $\omega_{Gr}$ and $\omega_{sat}$ coincide for $U_s = U_{Ts}$.

By setting $dA/dt=0$ in Equation (7), the saturation or limit-cycle amplitude $|A|^2_{sat}$ for $U_s < U_{Ts}$ is obtained as:

$$\omega_{Gi} - i \omega_{Gr} = [l_r + i l_i] |A|^2_{sat}$$ \hspace{1cm} (9a)

therefore,

$$|A|_{sat}^2 = \left[ \frac{\omega_{Gi}}{l_r} \right] \left[ \frac{1}{2} \right] = \left[ - \frac{\omega_{Gr}}{l_i} \right] \left[ \frac{1}{2} \right]$$ \hspace{1cm} (9b)

The modulus and the phase of $A$, $A=|A|\exp[-i\phi]$, into the S-L equation, Equation (7) becomes:

$$\frac{1}{|A|} \frac{d|A|}{dt} = \omega_{Gi} - l_r |A|^2 = \omega_{Gi} \left[ 1 - \frac{|A|^2}{|A|^2_{sat}} \right]$$ \hspace{1cm} (10a)
\[ \frac{d\phi}{dt} = -\omega_{Gr} - l|A|^2 = -\omega_{Gr} - \omega_{Gi} \left[ \frac{l_i}{l_r} \left\{ \frac{|A|^2}{|A_{sat}|^2} \right\} \right] \]  

(10b).

Equation (10) shows that the ratio \( l_i/l_r \) is a constant. Since it is related to a nonlinear frequency shift, the ratio is independent of the location where \( A \) is measured. When the amplitude \( |A| \) reaches the saturation amplitude \( |A_{sat}| \), the frequency \( d\phi/dt \) equals the saturated (measured) frequency, \( \omega_{sat} \), i.e. \( d\phi/dt = \omega_{sat} \). Equation (10b) becomes

\[ \omega_{Gr} - \omega_{sat} = -\omega_{Gi} \left[ \frac{l_i}{l_r} \right] \]  

(11).

By using Equations (11), (8a) and (8b), the ratio \( l_i/l_r \) can be obtained as:

\[ \left[ \frac{l_i}{l_r} \right] = \frac{\omega_{Gr} - \omega_{sat}}{\omega_{Gi}} = -\frac{\beta}{\alpha} \]  

(12).

Therefore, the coefficient of the S-L equation can be determined by the ratio of slopes between real and imaginary parts of the global frequency near the threshold suction speed. Figure 26 shows the distributions of real and imaginary parts of the global frequency at different suction speeds. The coefficient of the S-L equation is obtained as:

\[ \left[ \frac{l_i}{l_r} \right] \approx -0.57 \]  

(13)

for the plane wake with base suction. The real part of the global frequency, \( \omega_{Gr} \), and the measured vortex shedding frequency, \( \omega_{sat} \) (non-dimensionalized by \( U_\infty \) and \( D \)), intersect near the threshold suction speed. This is very close to the prediction from Equation (11), i.e., that \( \omega_{Gr} \) is equal to \( \omega_{sat} \) when \( \omega_{Gi} \) equals zero.

From the Stuart-Landau model, the nonlinear saturated frequency can be calculated from Eq. 12 as:
\[ \omega_{\text{sat}} = \omega_{\text{Gr}} + \omega_{\text{Gr}} \left( \frac{t_l}{t_c} \right) = \omega_{\text{Gr}} - \frac{\beta}{\alpha} \]  

(14)

By applying this non-linear frequency selection criteria, the data is plotted in terms of the Strouhal number, St, which is equal to \(2\pi fD/U_\infty\) (Fig. 25). Both the linear (Pierrehumbert 1984, Monkewitz & Nguyen 1987) and nonlinear stability analysis predict the trend of decreasing instability frequency with increasing suction speeds.

6-3 Sensitivity to External Perturbations

In a self-excited flow such as a wake, it is interesting to find out how the flow responds to the external disturbances, artificial perturbations or background noise. We will examine this problem in the frequency domain. The stabilizing rod placed inside the absolute instability region, \(X/D=0.5, \ Y/D=0\), serves as the forcing device to generate a small-amplitude perturbations in the absolutely unstable region of the wake flow. The excitation amplitude \(\ ? \ cm/sec\) is very small. The amplitude used in the present forcing studies is below the required threshold level for lock-in states (Barbi et al. JFM 170, p.527). The forcing frequency, \(f_f\), of the rod is close to the most unstable frequency or fundamental vortex shedding frequency, \(f_m\), of the wake. \(f_m\) is determined experimentally in a wake without suction as shown in Fig. 27a. The forcing frequencies, \(f_f\), were varied from 0.9 to 2.4 Hz, covering a range from about 1/2 \(f_m\) to 1.3 \(f_m\) in the tested range. The spectrum measured at a location fairly far downstream from the absolutely unstable region, \(X/D=5.0, \ Y/D=1.0\), represents the response of the unstable flow to the forcing. The spectral peak is defined as the response frequency, \(f_r\). When \(U_s/U_\infty=0\), the energy spectra at different forcing frequencies are plotted in Figure 27. In Fig. 27a, the peak frequency of the energy spectrum corresponds to the fundamental frequency in the unforced wake flow without suction, \(f_m\). When the control rod vibrator oscillates at 1.5
Hz and at 2.0 Hz, the spectra are shown in Figs. 27b and 27c, respectively. The response frequencies in both cases remain unchanged and the peak frequencies are the same as the fundamental frequency in Fig. 27a, i.e., the flow does not respond to the forcing. Similar results were found in the case of $U_s/U_\infty =0.25$, where the fundamental frequency is reduced to 1.61 Hz (Fig. 29). When the suction speed reaches $U_s/U_\infty =0.56$, unlike the sub-critical suction cases in Figure 27, the energy spectrum in the unforced wake flow with supercritical suction (Fig. 28a) does not have a clear peak at the most amplified frequency. The self-excited instability disappears and the wake flow becomes stable. The energy spectra at different forcing conditions start to change. Figs. 28b and 28c show that the spectral peaks are located at the same frequency as the forcing frequency. The wake flow responds to the forcing under the supercritical suction condition. The relationship between the response frequency and the forcing in this suction range is plotted in Figure 29.

These results clearly show the difference between a self-excited wake and a non-self-excited wake. When the wake is self-excited, the frequency is dominated by the intrinsic one determined by the absolute instability. The external perturbations cannot shift the wake frequency and even have no effect on the velocity spectra (Fig. 27). When the global instability is turned off, the artificial disturbance becomes detectable and convects in the streamwise direction even though the amplitude will not change in this neutrally stable flow (Fig. 23). This experiment provides clear evidence of the wake sensitivity to external noise in a wake.

**SUMMARY**

A base suction effectively reduces the size of the absolutely unstable region and forms a non-parallel flow region in the near wake. A threshold suction velocity is found and is
equals to approximately half free stream velocity. The threshold velocity can be accurately predicted by global stability analysis with the non-parallel correction. Below the threshold suction, the wake is dominated by self-excitation and is not sensitive to external disturbances. In the case of the supercritical suction, the non-parallel flow effect drives the global instability into the stable regime. *The entire flow is neutrally stable.* \(Vortex \text{ street is not observed.}\) Disturbances convect with the flow at 78% of the free stream velocity and without much change of the amplitude.

**ACKNOWLEDGMENT**

The authors appreciate Professor Peter Monkewitz’s valuable suggestions. This work is mainly supported by a grant from the Office of Naval Research. The early phase of the research was supported by the Air Force Office of Scientific Research.
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\( \frac{U_s}{U_\infty} = 0.5, U_\infty = 16.0 \text{ cm/s} \)
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$U=16\text{cm/s, } \frac{U_s}{U_0} = f = \frac{1}{1.8438\text{hz}}$

$U=16\text{cm/s, } \frac{U_s}{U_0} = 0.75, f = \frac{1}{1.7\text{hz}}$
Three Frequency selection criteria

$\omega_{oi}^{(D)}$

$X_I$: Initial Resonance (P. A. Monkewitz & L. N. Nguyen)

$X_p$: Maximum absolute growth rate (Pierrehumbert)

$X_k$: Transition point (Koch)

Figure 24 Sketch of Frequency selection criteria
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