Advanced Fluid Mechanics: Homework #5

1. Write down the complex potential for a source of strength \( m \) located at \( z = ih \) and a source of strength \( m \) located at \( z = -ih \). Show that the real axis is a streamline in the resulting flow field, and so deduce that the complex potential for the two sources is also the complex potential for a flat plate located along \( y = 0 \) with a source of strength \( m \) located a distance \( h \) above it.

Obtain the pressure on the surface of the plate mentioned above from the Bernoulli equation. Integrate this pressure over the entire surface of the plate, and so show that the force acting on the plate, due to the presence of the source, is \( \rho m^2/(4\pi h) \). Take the pressure below the plate to be equal to the stagnation pressure in the fluid.

2. Write down the complex potential for a source of strength \( m \) located at \( z = -b \), a source of strength \( m \) located at \( z = -a^2/b \), a sink of strength \( m \) located at \( z = a^2/b \), a sink of strength \( m \) located at \( z = b \), and a constant term \(-im/2\). Expand the result for small values of \( z/b \) and \( a^2/(bz) \), and hence show that if \( b \to \infty \) and \( m \to \infty \) in such a way that \( m/b \to \pi U \), the resulting complex potential is that of a uniform flow of magnitude \( U \) flowing past a circular cylinder of radius \( a \).

3. Obtain the complex potential for a source of strength \( m \) located at \( z = be^{i(a+\pi)} \), a source of strength \( m \) located at \( z = (a^2/b)e^{i(a+\pi)} \), a sink of strength \( m \) located at \( z = (a^2/b)e^{ia} \), a sink of strength \( m \) located at \( z = be^{ia} \), and a constant term of magnitude \(-im/2\). Expand this result for small values of \( z/b \) and \( a^2/(bz) \), and hence show that as \( b \to \infty \) and as \( m \to \infty \) such that \( m/b \to \pi U \), the circle of radius \( a \) is a streamline. Hence show that the complex potential for a uniform flow of magnitude \( U \) approaching a circular cylinder of radius \( a \) at an angle of attack \( \alpha \) to the horizontal is

\[
F(z) = U \left( ze^{-i\alpha} + \frac{a^2}{z} e^{i\alpha} \right)
\]
4. Determine the complex potential for a circular cylinder of radius $a$ in a flow field produced by a counterclockwise vortex of strength $\Gamma$ located a distance $l$ from the center of the cylinder. This may be done by writing the complex potential for the following system of singularities:

(a) A clockwise vortex of strength $\Gamma$ located at $z = -b$
(b) A counterclockwise vortex of strength $\Gamma$ located at $z = -a^2/b$
(c) A clockwise vortex of strength $\Gamma$ located at $z = a^2/l$
(d) A counterclockwise vortex of strength $\Gamma$ located at $z = l$
(e) A constant term of magnitude $-\left[\frac{i\Gamma}{(2\pi)}\right] \log b$

Then let $b \to \infty$ and show that the circle of radius $a$ is a streamline. Obtain the value of the force acting on the cylinder due to the vortex at $z = l$ by applying the Blasius law to a contour that includes the cylinder but excludes the vortex at $z = l$.

(25%) 

5. Go to the following webpage of MIT Open Course