Advanced Fluid Mechanics: Homework #2

1. Derive the net surface force per unit volume acting on the fluid particle is
\[ \frac{\partial \sigma_{ij}}{\partial x_j} \]
by using the differential control volume as shown below. (30%)

2. Say that we have a fluid of variable density at rest and acted upon by a conservative body force \( F_j \) (per unit mass). For a conservative body force field, \( F_j \) (per unit mass) can be expressed as
\[ F_j = \frac{\partial \varphi}{\partial x_j} = \nabla \varphi. \]

Derive the necessary relation between \( \rho \) and \( \varphi \) such that fluid is indeed at rest and no motion at all. (Hint: use the momentum equation eg. with \( u_j=0 \) to start with) (40%)

3. Write out the expression for the dissipation function \( \Phi \) in Cartesian coordinate \((x,y,z)\) with corresponding velocity component \((u,v,w)\).
\[ \Phi = \frac{\mu}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = \frac{\mu}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 \]
(30%)